



COURSE DESCRIPTION CARD - SYLLABUS

Course name

Calculus

Course

Field of study

Year/Semester

Computing

1/1

Area of study (specialization)

Profile of study

general academic

Level of study

Course offered in

First-cycle studies

polish

Form of study

Requirements

full-time

compulsory

Number of hours

Lecture

Laboratory classes

Other (e.g. online)

30

Tutorials

Projects/seminars

30

Number of credit points

6

Lecturers

Responsible for the course/lecturer:

Responsible for the course/lecturer:

Zbigniew Walczak, Ph.D.

email: zbigniew.walczak@put.poznan.pl

phone: 61 665 2812

Faculty of Control, Robotics and Electrical
Engineering

ul. Piotrowo 3A, 60-965 Poznań

Prerequisites

Basic mathematical knowledge from secondary school. Student is able to meet the challenges arising from the high school. Student has a ability to think logically. Student is aware of the need to expand their competences. He understands the need for learning.

Course objective

Basic knowledge in differential calculus, among others theory of functions of several variables, integral calculus, infinite series, power series and differential equations, which is necessary to study informatics and engineering sciences.



Course-related learning outcomes

Knowledge

1. The student knows the concepts of limit of the sequence and the function, continuity of the function, derivative and its applications, convergence or divergence of the series.
2. The student knows the concept of integrals and knows methods of calculation and applications.
3. The student has knowledge of partial derivatives, to be able to calculate extrema for functions of two variables and implicit function.
4. The student has knowledge of Taylor and Maclaurin series.
5. The student knows types of elementary differential equations and methods of their solving.

Skills

1. The student can define functions and he is able to describe their properties. He can evaluate limits of sequences and functions. Moreover, the student can apply theorems and methods of calculus in optimization problems. He is able to find the extrema and analyse of functions in order to construct the graph of them.
2. The student can calculate indefinite, definite and improper integrals, measures of areas, the length of curves, volumes and surface areas of solids of revolution.
3. Student is able to determine convergence or divergence of series.
4. Student is able to find a power series representation of the function and determine its convergence.
5. The student can recognize type of differential equation and solve it.

Social competences

1. Student is aware of the level of their knowledge in relation to research in exact and technical sciences.
2. Student is aware of the deepening and expansion of knowledge to solve newly created technical problems.

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Formative assessment:

- based on answers to questions related to subjects covered during former lectures and based on assessment of progress of implementation of assigned tasks,

Total assessment:

1. Lecture: assessment of knowledge and skills demonstrated in the written exam. The exam is assessed in a point system. The condition of passing the exam is obtaining at least 50% of points.



2. Tutorials: two written assignments carried out under the teacher's supervision and constant assessment during tutorials of oral tasks. The condition of receiving a positive grade from tutorials is obtaining at least 50% of points. Additional points can be obtained for activity during classes.

Programme content

Lecture:

1. FUNCTIONS AND THEIR PROPERTIES

- definition of function, domain and range of function, graph of function
- bounded function, increasing and decreasing functions, periodic function, even and odd functions, function composition
- surjection, injection, inverse function
- natural logarithm, cyclometric functions, hyperbolic function, area functions
- implicit function, parametric representation of curve, curves in polar coordinates

2. SEQUENCES

- definition of sequence
- bounded sequence, monotone sequence
- limit of sequence (including Euler's number), convergent and divergent sequences,
- theorems of sequences (including arithmetic rule, the squeeze theorem)

3. LIMITS AND CONTINUITY OF FUNCTIONS

- definitions of limits
- properties of limits theorems of limits (including arithmetic rule, the squeeze theorem)
- indeterminate forms
- continuous functions and their properties (including Weierstrass theorem and Darboux's theorem)
- discontinuous functions

4. DERIVATIVE OF FUNCTION

- definition of derivative, geometric interpretation of derivative, right and left-hand derivatives, theorems of derivatives, properties and rules for finding derivatives
- tangent and normal lines,



- increments and differentials
 - higher order derivatives
 - L'Hospital's rule
 - Taylor's and Maclaurin's formulas and applications
 - horizontal and vertical asymptotes
 - ekstrema of functions
 - the first derivative test
 - concavity and the second derivative test
 - applications of extrema
5. INDEFINITE INTEGRALS
- antiderivative of function, properties of integration (including integration by parts, change of variable)
 - integration of rational function, method of partial fractions
 - trigonometric substitutions, integration of some irrational functions
6. DEFINITE AND IMPROPER INTEGRALS
- definition of definite integral, Riemann sum, geometric interpretation of definite integral, fundamental theorem of integral calculus
 - properties of the definite integral (including integration by parts, change of variable)
 - applications of the definite integral (including area, volume and surface area of solids of revolution, length of curve)
 - improper integrals (integrals with infinite limits of integration, integrals with discontinuous integrands)
7. INFINITE SERIES AND POWER SERIES
- definition of infinite series, sum of the series, necessary condition for convergence, convergent or divergent infinite series
 - convergence tests (integral test, comparison test, alternating series test, ratio test, root test)
 - absolute and conditional convergence
 - power series, radius of convergence, interval of convergence



- power series representations of functions
 - Taylor and Maclaurin series and applications
8. FUNCTIONS OF SEVERAL VARIABLES
- domain and range, graph
 - limits
 - partial derivatives
 - increments and differentials
 - extrema of functions of two variables
 - extrema of implicit function
9. DIFFERENTIAL EQUATIONS
- definition of differential equation, partial solution, general solution, boundary value problem
 - homogeneous and nonhomogeneous first-order differential equations, separable equations, first-order linear differential equations, exact first-order equations
 - homogeneous and nonhomogeneous second-order linear equations with constant coefficients

Tutorials:

1. FUNCTIONS AND THEIR PROPERTIES
- definition of function, domain and range of function, graph of function
 - bounded function, increasing and decreasing functions, periodic function, even and odd functions, function composition
 - inverse function
 - natural logarithm, cyclometric functions, hyperbolic function, area functions
 - implicit function, parametric representation of curve, curves in polar coordinates
2. SEQUENCES
- bounded sequence, monotone sequence
 - limit of sequence (including Euler's number), convergent and divergent sequences,
 - theorems of sequences (including arithmetic rule, the squeeze theorem)
3. LIMITS AND CONTINUITY OF FUNCTIONS



- definitions of limits
- properties of limits theorems of limits (including arithmetic rule, the squeeze theorem)
- indeterminate forms
- continuous functions and their properties (including Weierstrass theorem and Darboux's theorem)

- discontinuous functions

4. DERIVATIVE OF FUNCTION

- definition of derivative, geometric interpretation of derivative, right and left-hand derivatives, theorems of derivatives, properties and rules for finding derivatives
- tangent and normal lines,
- increments and differentials
- higher order derivatives
- L'Hospital's rule
- Taylor's and Maclaurin's formulas and applications
- horizontal and vertical asymptotes
- extrema of functions
- the first derivative test
- concavity and the second derivative test
- applications of extrema

5. INDEFINITE INTEGRALS

- antiderivative of function, properties of integration (including integration by parts, change of variable)
- integration of rational function, method of partial fractions
- trigonometric substitutions, integration of some irrational functions

6. DEFINITE AND IMPROPER INTEGRALS

- fundamental theorem of integral calculus
- properties of the definite integral (including integration by parts, change of variable)



- applications of the definite integral (including area, volume and surface area of solids of revolution, length of curve)
- improper integrals (integrals with infinite limits of integration, integrals with discontinuous integrands)

7. INFINITE SERIES AND POWER SERIES

- sum of the series, necessary condition for convergence, convergent or divergent infinite series
- convergence tests (integral test, comparison test, alternating series test, ratio test, root test)
- absolute and conditional convergence
- power series, radius of convergence, interval of convergence
- power series representations of functions
- Taylor and Maclaurin series and applications

8. FUNCTIONS OF SEVERAL VARIABLES

- domain and range, graph
- partial derivatives
- increments and differentials
- extrema of functions of two variables

Teaching methods

1. Lecture: multimedia presentation accompanied with examples presented on the blackboard and with questions to the group of students
2. Tutorials: solving problems on the board, initiating discussion about the solutions

Bibliography

Basic

1. M. Gewert, Z. Skoczylas, *Analiza Matematyczna 1 i 2, Definicje, twierdzenia i wzory*, Oficyna Wydawnicza GiS, Wrocław 2019.
2. W. Kryszicki, L. Włodarski, *Analiza matematyczna w zadaniach cz. I i II*, Wydawnictwo Naukowe PWN, Warszawa 2015.
3. M. Gewert, Z. Skoczylas, *Analiza Matematyczna 1 i 2, Przykłady i zadania*, Oficyna Wydawnicza GiS, Wrocław 2018.
4. W. Żakowski, G. Decewicz, *Matematyka cz. I, Analiza matematyczna*, WNT, Warszawa 2017.



5. W. Żakowski, W. Kołodziej, Matematyka cz. II, Analiza matematyczna, WNT, Warszawa 2017.

6. W. Żakowski, W. Leksiński, Matematyka cz. IV, WNT, Warszawa 2012.

Additional

1. H. J. Musielakowie, Analiza matematyczna tom 1 i 2, Wydawnictwo Naukowe UAM, Poznań 1993.

2. J. Morchało, Z. Ratajczak, J. Werbowski, Matematyka, Równania różniczkowe zwyczajne w zastosowaniach, Wydawnictwo Politechniki Poznańskiej, Poznań 1992.

Breakdown of average student's workload

	Hours	ECTS
Total workload	150	6,0
Classes requiring direct contact with the teacher	60	2,5
Student's own work (literature studies, preparation for tutorials, preparation for tests/exam) ¹	90	3,5

¹ delete or add other activities as appropriate